

APPENDIX

Here are some of the standard relationships as they appear in the $(\uparrow\downarrow\sqrt{})$ system.

$\uparrow\downarrow\sqrt{}$ Nomenclature	& in log-exponent form	Calculator Form (TI-89)
1) $a\sqrt{b} = b\uparrow(1/a)$	$a\sqrt{b} = b^{1/a}$	$b^{(1/a)}$
2) $(a\uparrow b)\downarrow c = b \cdot (a\downarrow c)$ $a\downarrow(b\sqrt{c}) = b \cdot (a\downarrow c)$ $(a\uparrow b)\downarrow c = a\downarrow(b\sqrt{c}) = b \cdot (a\downarrow c)$ $a\uparrow(b\downarrow c) = (c\downarrow b)\sqrt{a}$ $a\uparrow(b\downarrow a) = (a\uparrow b)\downarrow a = b$ * $a\downarrow(b\sqrt{a}) = (a\downarrow b)\sqrt{a} = b$ *	$\log_c a^b = b \cdot \log_c a$ $\log_{b\sqrt{c}} a = (\log_c a / (\log_c c^{1/b})) = b \cdot \log_c a$ $\log_c a^b = \log_{b\sqrt{c}} a$ $a^{\log_c b} = \log_{b^c} \sqrt{a}$ $a^{\log_a b} = \log_a a^b = b$ $\log_{b\sqrt{a}} a = \log_b a^{\sqrt{a}} = b$ $(a\sqrt{b})^c = a\sqrt{(b^c)} = b^{c/a}$	$(b \cdot \ln(a)) / \ln(c)$ $(b \cdot \ln(a)) / \ln(c)$ " " $a^{(\ln(b) / \ln(c))}$ $a^{(\ln(b) / \ln(a))}$
3) $(a\sqrt{b})\uparrow c = a\sqrt{(b\uparrow c)}$ * $= b\uparrow(c/a)$ $a\sqrt{(b\uparrow a)} = (a\sqrt{b})\uparrow b = b$ *	$(a\sqrt{b})^c = a\sqrt{(b^c)} = b^{c/a}$	$b^{(c/a)}$
4) $a\downarrow b = \frac{a\downarrow c}{b\downarrow c}$ $a\downarrow b = \frac{1}{b\downarrow a}$ $(1/a)\downarrow b = -(a\downarrow b) = a\downarrow(1/b)$	$\log_b a = \frac{\log_c a}{\log_c b}$ $\log_b a = \frac{1}{\log_a b}$ $\log_b(1/a) = -\log_b a = \log_{1/b} a$	$\frac{\ln(a)}{\ln(b)}$ " " $\ln(1/a) / \ln(b)$
5) $b\downarrow b = 1$ $1\downarrow b = 0$ $b\downarrow 1 = \pm\infty$ $0\downarrow b = -\infty$ $b\downarrow 0 = 0$	$\log_b b = 1$ $\log_b 1 = 0$	$\ln(b) / \ln(b) = 1$ $\ln(1) / \ln(b) = 0$
6) $ab\uparrow c = (a\uparrow c) \cdot (b\uparrow c)$ $a\uparrow(bc) = (a\uparrow b)\uparrow c$ $a\uparrow(b+c) = (a\uparrow b) \cdot (a\uparrow c)$ $(a+b)\uparrow c = \text{binomial expansion}$	$(ab)^c = (a^c) \cdot (b^c)$ $(a^b)^c = a^{bc}$ $a^{b+c} = a^b \cdot a^c$ $(a+b)^c$	$(a^c) \cdot (b^c)$ $(a^b)^c$
7) $(ab)\downarrow c = a\downarrow c + b\downarrow c$ $a\downarrow(bc) = 1 / (bc\downarrow a) = 1 / (b\uparrow a + c\uparrow a)$ $(a/b)\downarrow c = a\downarrow c - b\downarrow c$ $(1/b)\downarrow c = -(b\downarrow c)$	$\log_c ab = \log_c a + \log_c b$ $\log_{bc} a = 1 / (\log_a bc) = 1 / (\log_a b + \log_a c)$ $\log_c(a/b) = \log_c a - \log_c b$ $\log_c(1/b) = -\log_c b$	$\ln(ab) / \ln(c)$ $\ln(a/b) / \ln(c)$
8) $(ab)\sqrt{c} = a\sqrt{(b\sqrt{c})} = b\sqrt{(a\sqrt{c})}$ $a\sqrt{(b \cdot c)} = (a\sqrt{b}) \cdot (a\sqrt{c})$	$ab\sqrt{c} = a\sqrt{(b\sqrt{c})} = b\sqrt{(a\sqrt{c})}$ $a\sqrt{(b \cdot c)} = (a\sqrt{b}) \cdot (a\sqrt{c})$	$c^{(1/ab)}$ $(b \cdot c)^{(1/a)}$

* These 4 formulae are associative. No parentheses are required

Also notice that in every formulation in 2) & 2a), the symbols always occur in the sequence $\sqrt{}\downarrow\downarrow\sqrt{}$

Appendix (continued)

From the right hand column above (Calculator Form (TI-89)), you can see how versatile the hand held calculator can be. To perform the operations listed at far right, you (almost always) must perform several operations by hand to get the the problem in a form the calculator can recognize. If the operations \downarrow and $\sqrt{}$ were added to calculator (\uparrow is already there), the versatility and convenience of the calculator would be greatly enhanced. And this would not require any additional memory in the calculator. The same straightforward calculations, done by hand, that now must precede these operations, would be programmed into the calculator.*

That is to say, in order to presently use the hand held calculator for the \downarrow and $\sqrt{}$ operators, you must memorize the formulations in the left hand column.

* In particular

$$a \downarrow b = \ln(a)/\ln(b) \quad (\#4 \text{ in Appendix})$$

$$a \sqrt{b} = b^{(1/a)} \quad (\#1 \text{ in Appendix})$$

$$\begin{aligned} (a \uparrow b) \downarrow c &= \log_c a^b \\ &= b \cdot \log_c a \quad (\#2 \text{ in Appendix}) \\ &= b \cdot \ln(a)/\ln(c) \quad (\#4 \text{ “ “ ”}) \end{aligned}$$

$$a \uparrow (b \downarrow a) = (a \uparrow b) \downarrow a = b \quad (\#2 \text{ in Appendix})$$